Roll No. ....

(07/22-II)

5181

B.A./B.A. (Hons.)/B.Sc. EXAMINATION

(Second Semester)

MATHEMATICS

BM-123

Vector Calculus

Time : Three Hours *Maximum Marks :* $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} : 27 \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Find the constant a such that the vectors $2\hat{i} - j + \hat{k}$, $\hat{i} + 2j - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar. 2(1)

(b) If $\phi(x, y, z) = 3x^2y - y^3z^2$, find grad$\phi$ at the point $(1, -2, -1)$. 1.5(1)

(c) If $\vec{A}$ is any vector function, then prove that $\nabla \cdot \left(\nabla \times \vec{A}\right) = 0$. 1.5(1)

(d) The acceleration of a particle at any time $t \geq 0$ is given by :

$$\vec{a} = 12\cos 2t\,\hat{i} - 8\sin 2t\,\hat{j} + 16t\,\hat{k}.$$

Find velocity $\vec{v}$ at any time if $\vec{v}(0) = \vec{0}$.

1.5(1)

(e) Describe the co-ordinate surfaces for spherical co-ordinates. 1.5(1)

Section I

2\. (a) If :

$$\vec{A} = x_1\,\vec{a} + y_1\,\vec{b} + z_1\,\vec{c},$$

$$\vec{B} = x_2\,\vec{a} + y_2\,\vec{b} + z_2\,\vec{c},$$

$$\vec{C} = x_3\,\vec{a} + y_3\,\vec{b} + z_3\,\vec{c}$$

then prove that :

$$\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & x_3 & z_3 \end{vmatrix} \left(\vec{a} \cdot \left(\vec{b} \times \vec{c}\right)\right).$$

4(3)

(b) Prove that :

$$\left[\vec{a'}\ \vec{b'}\ \vec{c'}\right] = \frac{1}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]},$$

where $\vec{a'}$, $\vec{b'}$, $\vec{c'}$ is reciprocal system of $\vec{a}$, $\vec{b}$, $\vec{c}$. 4(2.5)

3. (a) Find the unit tangent vector to the curve $x = t^2 + 1,\quad y = 4t - 3,\quad z = 2t^2 - 6t$ at $t = 2$. 4(3)

(b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, −1, 2) in the direction $2\hat{i} - 3\hat{j} + 6\hat{k}$. 4(2.5)

Section II

4. (a) Find an equation for the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at the point (1, – 1, 2). 4(3)

(b) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r = |\vec{r}|$,

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 4(2.5)

5. (a) Find the constants a, b, c so that the vector :

$$\vec{V} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

is irrotational. 4(3)

(b) Prove that $\operatorname{curl}(\phi \operatorname{grad}\phi) = \vec{0}$. 4(2.5)

Section III

6. Express the velocity and acceleration of a particle in cylindrical co-ordinates. 8(5.5)

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7. (a) Express $\operatorname{curl} \vec{A}$ in orthogonal co-ordinates. 4(3)

(b) Represent the vector $\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in spherical co-ordinates and determine A_r, A_θ, A_ϕ. 4(2.5)

Section IV

8. (a) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$. 4(3)

(b) If $\vec{F}\left(2x^2 - 3z\right)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, evaluate $\iiint\limits_V \nabla \cdot \vec{F}\, dV$, where V is the closed region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$. 4(2.5)

9. Verify the divergence theorem for $\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. 8(5.5)