Roll No. ....

(01/22-II)

5159

B.A/B.A.(Hons.)/B.Sc. EXAMINATION

(First Semester)

MATHEMATICS

BM-111

Algebra

Time : Three Hours *Maximum Marks :* B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Prove the the adjoint of a non-singular matrix is non-singular. 2(2)

(b) The determinant of a unitary matrix has absolute value 1. 2(1)

(c) Apply Descartes' rule of sings to discuss the nature of the roots of the equation $3x^4+12x^2+5x-4=0$. 2(1)

(d) Write the quadratic form corresponding to the matrix : 2(1)

$$A=\begin{bmatrix} 2 & 1 & 2 \\ -3 & -3 & -1 \\ 4 & 1 & 3 \end{bmatrix}$$

Unit I

2. (a) Every square matrix A can be expressed in one and only one way as P + iQ, where P and Q are Hermitian matrices. 4(3)

(b) Reduce the matrix :

$$A=\begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

to the normal form and hence determine its rank. 4(2½)

3. (a) Determine the characteristic roots and corresponding characteristic vectors of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$. 4(3)

(b) Find the minimal polynomial of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ and show that it is non-derogatory. 4(2½)

Unit II

4. (a) For what value of λ, does the system

$$\begin{bmatrix} 1 & -2 & -1 \\ 3 & -1 & 2 \\ 0 & 1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$ has :

(i) no solution

(ii) a unique solution

(iii) more than one solution ? 4(3)

(b) Find the value of k such that the system of equations :

$$x + ky + 3z = 0$$
$$4x + 3y + kz = 0$$
$$2x + y + 2z = 0$$

has a non-trivial solution. 4(2½)

5. (a) Reduce the quadratic form $x_1^2 - 2x_2^2 + 3x_3^2 - 4x_2x_3 + 6x_3x_1$ to canonical form. Also, find the rank, index, signature and equations of transformation. 4(3)

(b) Prove that :

$$5x^2 + 26y^2 + 10z^2 + 4yz + 14zx + 6xy$$

is positive semi-definite. 4(2½)

Unit III

6. (a) Solve the equation :

$$x^4 - 8x^3 + 14x^2 + 8x - 15 = 0,$$

roots being in A.P. 4(3)

(b) Find the common roots of the equations :

$$x^4+3x^3-5x^2-6x-8=0$$

and

$$x^4+x^3-9x^2+10x-8=0.$$

Hence solve them completely. 4(2½)

7. (a) Solve the equation

$$15x^4-8x^3-14x^2+8x-1=0,$$

given that the roots are in H.P. 4(3)

(b) Find the equation of squared differences of the roots of the equation $x^3+3x+2=0$ and show that the given equation has a pair of imaginary roots. 4(2½)

Unit IV

8. (a) Solve the equation $x^3-12x-65=0$ by Cardan's method. 4(3)

(b) Solve by the method of resolution into quadratic factors : 4(2½)

$$x^4-3x^2-42x-40=0.$$

9. (a) Solve the equation

$$2x^4 + 6x^3 - 3x^2 + 2 = 0$$

by Ferrari's method. 4(3)

(b) Show that the equation

$$x^5 + 5x^2 + 3x + k = 0$$

has at least two imaginry roots for all values of k. 4(2½)