Roll No. ....

(01/22-II)

5160

B.A./B.A. (Hons.)/B.Sc. EXAMINATION

(First Semester)

MATHEMATICS

BM-112

Calculus

Time : Three Hours *Maximum Marks* : B.Sc. : 40, B.A. : 26

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. All questions carry equal marks.

1. (i) Show by definition that the function $f(x) = 3x + 4$ has limit 10 when $x \to 2$.

(ii) Expand $\sin^{-1} x$ in powers of x by Maclaurin's series.

(iii) Define asymptote and give *two* different forms of definition of asymptote.

(iv) Define the concavity and convexity of a curve.

(v) State Pappu's and Guldin's theorem for volume of revolution.

Section I

2. (i) Using $\varepsilon - \delta$ definition, prove that $\cos^2 x$ is a continuous function.

(ii) Show that the function :

$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous and derivable at $x = 0$, but its derivative is not continuous at $x = 0$.

3. (i) If $y = \left(\sin^{-1} x\right)^2$, prove that :

$$\left(1 - x^2\right) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0.$$

(ii) Expand $\tan x$ in powers of $x-\frac{\pi}{4}$ upto first four terms.

Section II

4. (i) Find all the asymptotes of the curve :

$$x^3+3x^2y-4y^3-x+y+3=0.$$

(ii) Find the asymptotes of the curve :

$$r\cos\vartheta = a\sin^2\vartheta.$$

5. (i) If ρ_1 and ρ_2 are the radii of curvature at the extremities of a pair of semi-conjugate diameters of an ellipse, prove that $\left[(\rho_1)^{\frac{2}{3}}+(\rho_2)^{\frac{2}{3}}\right](ab)^{\frac{2}{3}} = a^2+b^2$.

(ii) Find the radius of curvature for the curve $r^n = a^n \cos n\vartheta$.

Section III

6. Trace the curve $r = a(1 = \sin\vartheta)$.

7. (i) Find the reduction formula for $\int \sin^n x.dx$ and hence evaluate $\int \sin^4 x.dx$.

(ii) Find the entire length of the cardioid $r = a(1-\cos\vartheta)$, and show that the arc of the upper half of the cardioid is bisected by $\vartheta = \frac{2\pi}{3}$.

Section IV

8. (i) Find the area common to the parabola $y^2 = 4ax$ and $x^2 = 4ay$.

(ii) Find the area of a loop of the curve $x^3 + y^3 = 3axy$.

9. (i) The loop of the curve $2ay^2 = x(x-a)^2$ revolves about x-axis. Find the volume of the solid so formed.

(ii) Find the surface of the right circular cone formed by the revolution of a right-angled triangle about a side which contains the right angle.