Roll No. .....

(01/22-II)

5161

B.A/B.A.(Hons.)/B.Sc. EXAMINATION

(First Semester)

MATHEMATICS

BM-113

Solid Geometry

Time : Three Hours *Maximum Marks :* B.Sc. : 40, B.A. : 27

Note : There are nine questions in all. Candidates are required to attempt *five* questions, selecting *one* question from each Section and the compulsory question consisting of five parts distributed over all the four Sections.

(Compulsory Question)

1. (a) Find the equation of the tangent to the conic, $x^2 + 2xy + y^2 - 2x - 1 = 0$ at (0, 1). 1½(1)

(b) To find the condition that the cone

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0,$$

may have three mutually perpendicular generators. 1½(1)

(c) Define normal to a conicoid. To find equation of the normal at the point (x_1, y_1, z_1) of the conicoid $ax^2 + by^2 + cz^2 = 1$. 1½(1)

(d) Define diametral plane. 2(1)

(e) Prove that one conicoid confocal with a given ellipsoid touches a given plane. 1½(1)

Section I

2. Trace the conic $2x^2 + 3xy - 2y^2 - 7x + y - 2 = 0$ and calculate the eccentricity of the conic. 8(5½)

3. (a) Show that the product of the normals drawn from a point ρ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{\lambda_1\lambda_2(\lambda_1-\lambda_2)}{(a^2-b^2)}$, where λ_1, λ_2 are parameters of confocals through p. 4(3)

(b) In any conic, prove that the sum of thereciprocal of the two perpendicular focal chords is constant. 4(2½)

Section II

4. (a) Find the equations of circle, circumscribing the triangle formed by the three points $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ and obtain the co-ordinates of the centre of the circle. 4(3)

(b) Find the equation of the sphere, which touches the sphere $x^2 + y^2 + z^2 - z + 3y + 2z - 3 = 0$ at the point $(1, 1, -1)$ and passes through the origin. 4(2½)

5. (a) Prove that :

$$x^2 - y^2 + z^2 - 2x + 4y + 6z + 6 = 0$$

represents a right circular cone, whose vertex is the point (1, 2, –3), axis is parallel to OY and semi-vertical angle is 45°. 4(3)

(b) Find the equation of the cylinder which intersects the curve :

$$ax^2 + by^2 + cz^2 = 1,\ lx + my + nz = p$$

and whose generators are parallel to z-axis. 4(2½)

Section III

6. (a) Prove that the sum of the squares of the reciprocals of any *three* mutually perpendicular diameters of an ellipsoid is constant. 4(3)

(b) To prove that six normals can be drawn from a given point to the ellipsoid. 4(2½)

7. (a) Find the locus of the straight lines drawn through a fixed point (α, β, γ) at right angles to its polars w.r.t. the conicoid $ax^2 + by^2 + cz^2 = 1$. 4(3)

(b) Find the locus of the centres of sections the conicoid $ax^2 + by^2 + cz^2 = 1$, which touch the conicoid $\alpha x^2 + \beta y^2 + \gamma z^2 = 1$. 4(2½)

Secton IV

8. (a) Prove that the section of the conicoid $ax^2 + by^2 + cz^2 = 1$ by a tangent plane to the cone $\frac{x^2}{b+c} + \frac{y^2}{c+a} + \frac{z^2}{a+b} = 0$ is a rectangular hyperbola. 4(3)

(b) Find the equations to the generators of the paraboloid $(x + y + z)(2x + y - z) = 6z$ which pass through the point (1, 1, 1). 4(2½)

9. (a) A given plane and parallel tangent plane to a conicoid are at distance p and p_0 from the centre. Prove that the parameter of the confocal conicoid which touches the plane is $(p_0^2 - p^2)$. 4(3)

(b) Show that the surface represented by the equation :

$$26x^2 + 20y^2 + 10z^2 - 4yz - 16zx - 36xy + 52x - 36y - 16z + 25 = 0$$

is an elliptic cylinder. Find the equation of its axis. 4(2½)