Roll No. .....

(01/22-II)

5200

B.A./B.A. (Hons.)/B.Sc. EXAMINATION

(Third Semester)

MATHEMATICS

BM-232

Partial Differential Equations

Time : Three Hours *Maximum Marks :* B.Sc. : 40, B.A. : 26

Note : The candidates are required to attempt *five* questions in all, selecting the compulsory Q. No. 1 and *one* question from each Unit. Marks in brackets are for B. A. students.

Compulsory Question

1. (a) Define singular solution and general solution of PDE. 2(2)

(b) Solve $\left(D^4 + D'^4\right)z = 0$. 1½(1)

(c) Classify $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$. 1½(1)

(d) Define Laplace equation and wave equation. 1½(1)

(e) Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$. 1½(1)

Unit I

2. (a) Find the partial differential equation of all spheres whose centre lie on z-axis. 4(2½)

(b) Solve $\left(x^2 + 2y^2\right)p - xyq = xz$. 4(2½)

3. (a) Find complete integral of $x(1+y)p = y(1+x)q$. 4(2½)

(b) Find complete integral of $px + qy = pq$ using Charpit's method. 4(2½)

Unit II

4. (a) Solve :

$$\left(D^3 - 4D^2D' + 5DD'^2 - 2D'^3\right)z = e^{2x+y}.$$

4(2½)

(b) Solve :

$$\left(D^2 - DD' + D' - 1\right)z = \cos(x + 2y).$$

4(2½)

5. (a) Solve :

$$\left(x^2D^2 + 2xyDD' + y^2D'^2\right)z = x^2y^2.$$

4(2½)

(b) Solve :

$$\left(3D^2 - 2D'^2 + D' - 1\right)z = 4e^{x+y}\cos(x + y).$$

4(2½)

Unit III

6. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. 8(5)

7. Solve : $x^{-2}r + y^{-2}t = x^{-3}p - y^{-3}q$. 8(5)

Unit IV

8. Solve the Cauchy problem for the equation $\frac{\partial^2 z}{\partial x^2} - \frac{1}{C^2} \cdot \frac{\partial^2 z}{\partial t^2} = 0$, $C > 0$ subject to the conditions $z(x,0) = f(x)$ and $\left[\frac{\partial z}{\partial t}\right]_{t=0} = g(x)$. 8(5)

9. (a) Solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ with the conditions :

$u(0,t) = u(l,t) = 0$, $u(x,0) = x(l-x)$. 4(2½)

(b) Describe the method of separation of variables to find the solution of Laplace equation. 4(2½)